

paths starting with widely different initial conditions. This strongly suggests that this state of motion is stable. Probably it is unique for ordinary launchings. It is plausible that each boomerang has a characteristic final state of motion. For instance, flight path calculations made for boomerang 195.1 (level 3) yielded a final state characterized by the values:

$$f = 9.2 \text{ rev/s}, V = 13.5 \text{ m/s}, U = 0.78, \Psi = 9.2^\circ, \theta = 61^\circ, \psi = 161^\circ,$$

$$\phi = 18.4, \Delta Z = 22.1 \text{ m}, \Delta t = 4.7 \text{ sec.}$$

Necessary conditions for a boomerang's final state of motion are:

$$\dot{V} = 0, \dot{\omega}_z = 0, \dot{\Psi} = 0, \dot{\theta} = 0, \dot{\psi} = 0. \quad (28.1)$$

These are 5 equations for the 5 unknowns $V, \omega_z, \Psi, \theta, \psi$. The equations can be solved by putting the right-handed sides of the equations (4.8) and (4.9) equal to zero. If (28.1) is satisfied, $\dot{\phi} = \text{constant}$ according to (4.9), and $\dot{Z} = \text{constant}$ while \dot{X} and \dot{Y} vary harmonically with ϕ according to (4.10). We shall not discuss the conditions for the existence, the stability, or the uniqueness of a boomerang's final state of motion.

In the boomerang flight presented in this section, the path curves counterclockwise (ϕ increases) until $t = 6.2 \text{ sec.}$ ($Z = 4.6 \text{ m}$); after this ϕ decreases and the path curves clockwise. In other cases this reversal may occur at an earlier or at a later instant. This phenomenon can often be observed in real boomerang flights. Sometimes a boomerang may be well on its way to its final state of motion before it hits the ground. Curiously, the first counterclockwise loop, so characteristic for the flight of a right-handed boomerang, may be considered as merely a transient form of motion, soon leading up to the boomerang's final state.

In the final state of motion discussed above, the gravitational force plays a crucial role. What would happen in the absence of gravity? In the following we shall suppose that $g = 0$, that the air is homogeneous (with density $\mu = 1.20 \text{ kg/m}^3$), and that there is no wind. The equations of motion (4.8), (4.9), (4.10) now are modified in an important aspect: for a given boomerang $\dot{V}, \dot{\omega}_z$ and $\dot{\Psi}$ depend only on the aerodynamic forces, and hence (in our model) only on V, ω_z and Ψ . The shape of the boomerang's flight path is independent of $\theta_0, \phi_0, \psi_0, X_0, Y_0, Z_0$. The path in

(Ψ, U) -space traversed by the boomerang depends only on Ψ_0 and U_0 , while V_0 (or f_0) determines the velocity at which this path is traversed.

A necessary condition for Ψ being stationary is given by (22.1), and for U being stationary by (22.2). The restriction $\vartheta = \frac{1}{2}\pi$, $\psi = 0$ is not relevant now because gravity is absent. The boomerang will travel to a point with $\dot{\Psi} = 0$, $\dot{U} = 0$: (Ψ^*, U^*) (under conditions not investigated by us). Because the boomerang continually loses kinetic energy V and ω_z (or f) decrease. Since $\dot{V}(\cdot) - V^2$, V decreases inversely with time, and the boomerang's path length increases logarithmically with time. The boomerang ends up moving, slower and slower, along a helical path, away to infinity.

A gravitationless flight for boomerang 101.1 was computed with the initial conditions: $f_0 = 10$ rev/s, $V_0 = 25$ m/s, $\Psi_0 = 0^\circ$. This boomerang's final motion is characterized by:

$$\Psi^* = 2.0^\circ, U^* = 0.86$$

and the helical path by:

$$\phi = 34 \text{ m}, \Delta R = 31 \text{ m}, \Delta P = 112 \text{ m}, \Delta n = 70$$

where ϕ is the diameter of the helix, ΔR is the difference in the boomerang's position, ΔP the path length and Δn the number of revolutions between two points one turn of the helix apart. The helix is traversed clockwise as seen from the point of launching. The direction of the helix' axis depends on ϑ_0 , ϕ_0 and ψ_0 . The helical motion begins rather soon after the start: already at $t = 7$ sec. we have $\Psi = 2.0^\circ$, $U = 0.86$. At $t = 18.3$ sec. the boomerang has completed 100 spin revolutions and $f = 2.9$ rev/s, $V = 4.7$ m/s. At $t = 94.1$ sec. 200 revolutions are completed and $f = 0.7$ rev/s, $V = 1.1$ m/s.

The relative simple shape of the flight path in the gravitationless case is due to the fact that the motion proceeds at constant values for Ψ and U . In the presence of gravity there is no fixed equilibrium point (Ψ^*, U^*) , and the boomerang moves about in the (Ψ, U) -plane. Much of the charm of real boomerang flights originates from the interplay of aerodynamic and gravitational forces.

§29 Wind.

In the practice of boomerang throwing the influence of wind on the boomerang's flight can be an important factor. Even a very light breeze of 1 m/s may significantly alter the shape of a flight path. In the absence of wind it does not matter whether the boomerang is thrown to the south or to the north; the flight path will have the same shape. As soon as there is wind, the direction in which the boomerang is launched does matter, and the angle φ_0 may be a very important parameter indeed. If the wind is too strong, a boomerang's performance may be spoilt, the boomerang is taken with the wind and invariably lands at a considerable distance downwind. On the other hand, a moderate breeze may assist a boomerang in coming back. After carrying out a few trial throws in which the angle φ_0 is varied, the boomerang thrower may obtain perfect return flights. This is particularly pleasing if the boomerang in case is incapable of returning well under windless conditions. Usually the optimum launching conditions are such that $\beta - \varphi_0 \approx 45^\circ$ (β determines the wind's direction, see §8).

Two such cases are shown in fig. 29.1 and fig. 29.2. The 4 flight paths are computed for boomerang 245.1 (level 3), which is associated with boomerang L5, see §18. For the flights of fig. 29.1 the initial conditions are:

$$f_0 = 10 \text{ rev/s}, V_0 = 25 \text{ m/s}, \psi_0 = 0,$$

$$\theta_0 = 80^\circ, \varphi_0 = 0^\circ, \psi_0 = -20^\circ,$$

$$X_0 = 0 \text{ m}, Y_0 = 0 \text{ m}, Z_0 = 1.8 \text{ m},$$

a) no wind,

$$\text{b) } \beta = 45^\circ, W_0 = 1.3 \text{ m/s}, W_1 = 0.6 \text{ m/s/10 m}.$$

In the flights of fig. 29.2 the initial conditions are the same, except that $\theta_0 = 70^\circ, \psi_0 = -30^\circ$.

	wind	t	P	n	D	Z ₂	D ₂	D ₁	f ₂	f ₁	V' ₁	ψ' ₂	U' ₁
29.1	no	10.1	112.6	98.5	17.6	13.4	44.1	15.7	10.6	9.0	2.0	80.9	.10
	yes	9.9	121.9	94.6	1.2	15.9	41.1	0.9	10.3	8.9	2.9	80.1	.09
29.2	no	12.2	114.2	122.1	24.1	16.1	39.2	20.5	11.1	9.0	2.4	64.8	.11
	yes	13.3	122.0	134.0	4.3	18.0	34.0	2.6	11.3	9.0	2.1	52.8	.13

table 29.1. Some values for the flights of fig. 29.1 and 29.2.

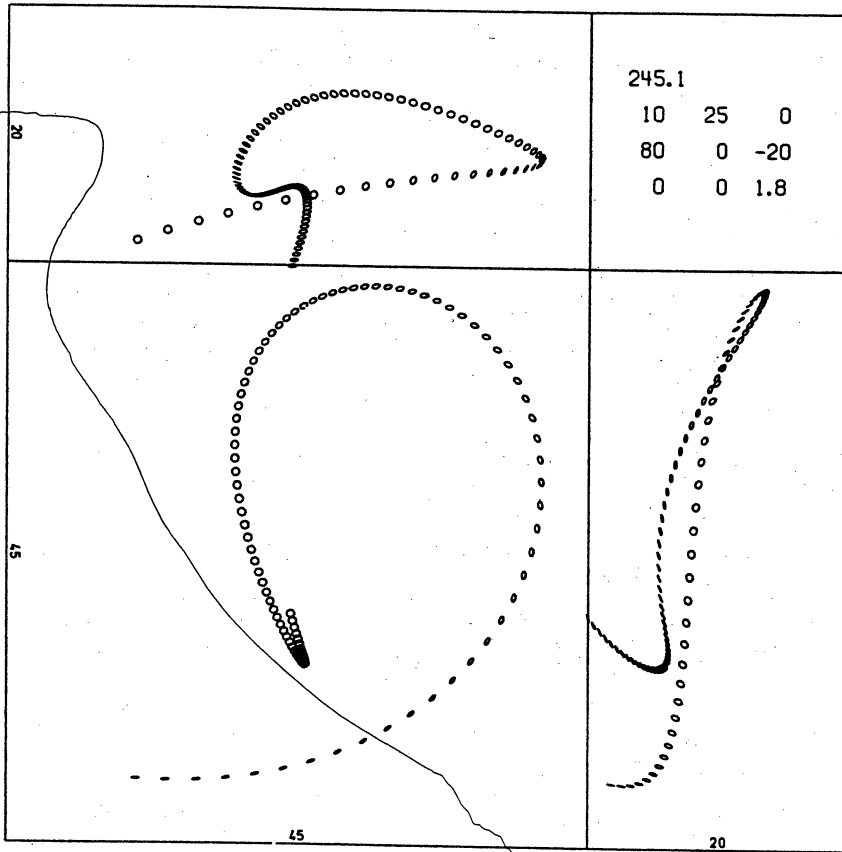


fig. 29.1a

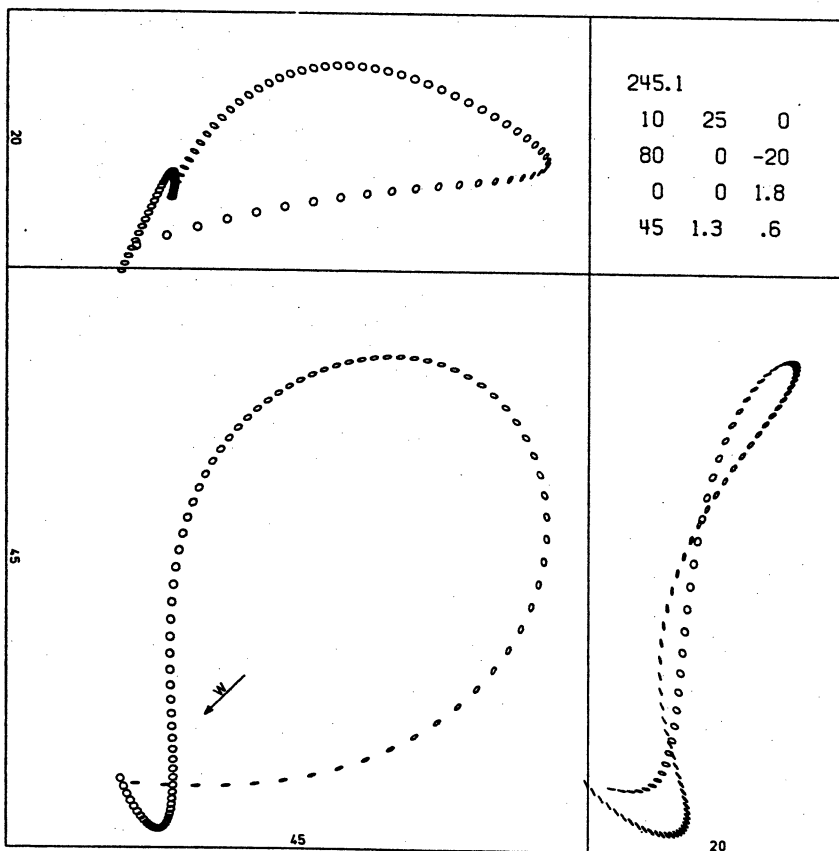


fig. 29.1b

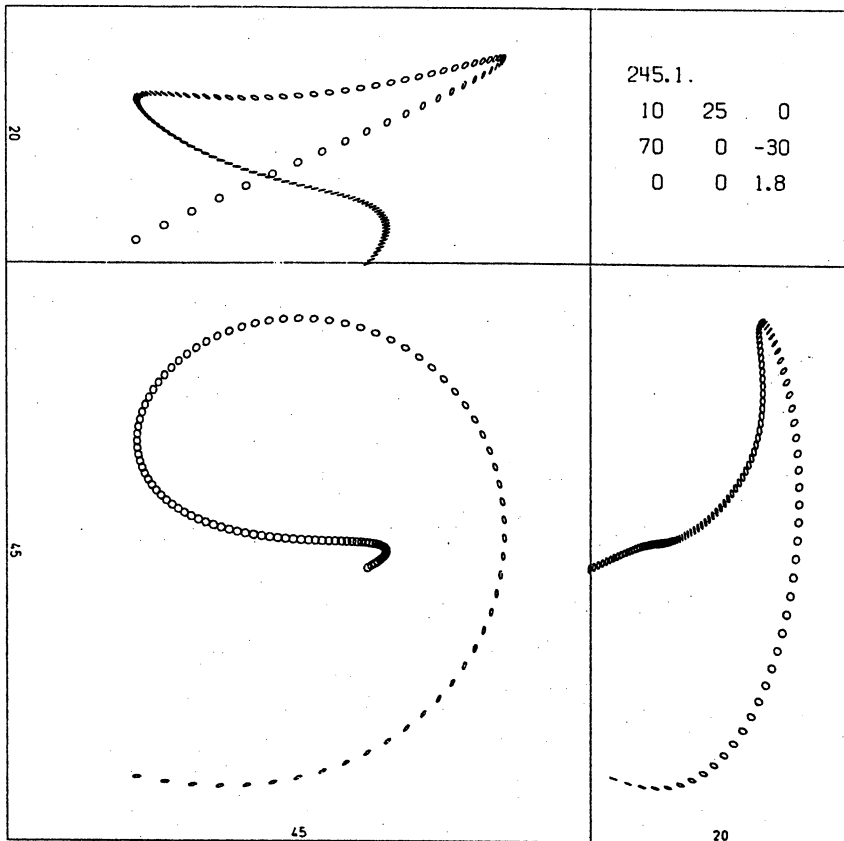


fig. 29.2a

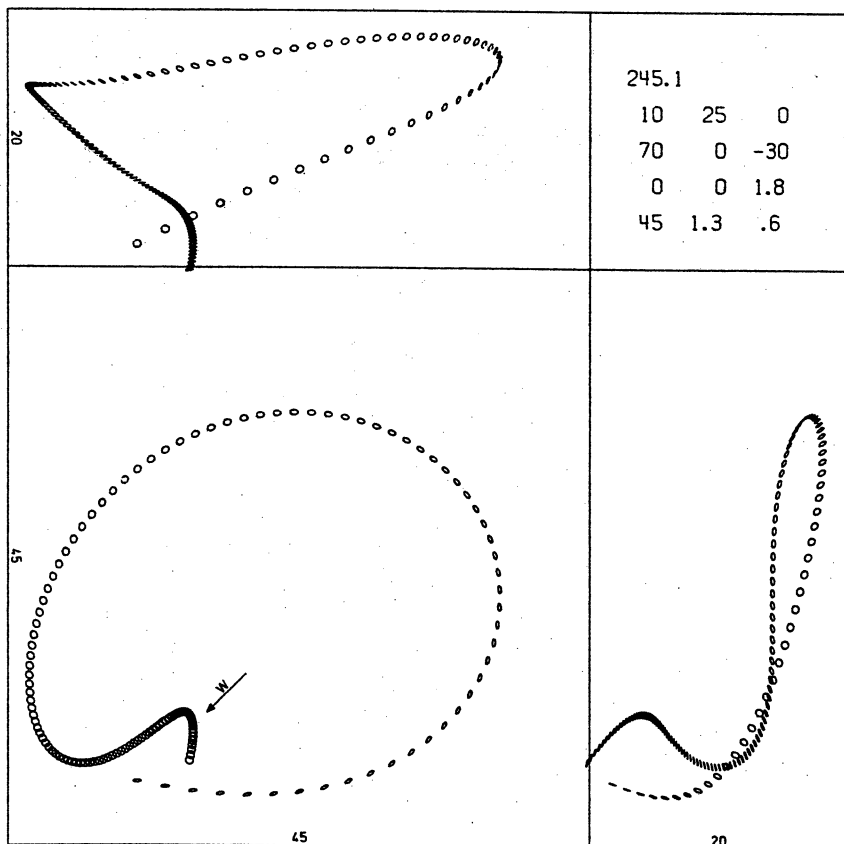


fig. 29.2b

Some values for these 4 flights are listed in table 29.1. In these examples the light breeze helps the boomerang to return beautifully. It is rather unfortunate that the field experiments with boomerang L5 (see §18) did not produce level 1 flight paths suitable for determining the influence of wind.

The main influence of the wind on a boomerang's flight can be explained as follows. Suppose the wind velocity field is constant and homogeneous ($W_1 = 0$). Use a coordinate system (X', Y', Z') moving with the air. In this inertial frame the air is motionless. However, the initial conditions for the boomerang's flight are modified: V_0 , Ψ_0 and U_0 have to be replaced by V'_0 , Ψ'_0 and U'_0 according to §8. For instance, if a boomerang is launched directly into the wind, $V'_0 > V_0$ and $U'_0 > U_0$. In the (X', Y', Z')-system the boomerang traverses an ordinary flight path, starting with the modified initial conditions. This trajectory can be transformed back to the old (X, Y, Z)-system according to:

$$\left. \begin{aligned} X(t) &= X'(t) + tW\cos\beta \\ Y(t) &= Y'(t) + tW\sin\beta \\ Z(t) &= Z'(t) \end{aligned} \right\} (29.1)$$

where W is the constant wind speed. If W is very small, the modification of the initial conditions has little influence, and the main effect of the wind is that the velocity of the air is added to the velocity of the boomerang. If the flight lasts for a time T , the landing point is shifted downwind over a distance $\approx WT$. In reality the wind speed varies in space and time, but the above line of reasoning usually is qualitatively valid.

In fig. 29.3a through h flight paths are shown for boomerang 101.1 (level 2), which are computed with the initial conditions:

$$\begin{aligned} f_0 &= 10 \text{ rev/s}, V_0 = 25 \text{ m/s}, \Psi_0 = 0^\circ, \\ \theta_0 &= 80^\circ, \varphi_0 = 0^\circ, \psi_0 = -10^\circ, \\ X_0 &= 0 \text{ m}, Y_0 = 0 \text{ m}, Z_0 = 1.8 \text{ m}, \\ \beta &= \text{variable}, W_0 = 2 \text{ m/s}, W_1 = 1 \text{ m/s/10 m.} \end{aligned}$$

The values chosen for β are respectively: $0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ$.

β	V'_0	U'_0	Ψ'_0	D	Z_2
none	25.0	1.34	0.0	7.7	9.9
0°	27.1	1.45	0.0	27.1	13.0
45°	26.5	1.42	-3.3	20.0	12.6
90°	25.0	1.34	-4.9	14.6	10.6
135°	23.5	1.25	-3.7	17.9	8.2
180°	22.9	1.22	0.0	22.1	6.8
225°	23.6	1.26	3.7	25.0	7.4
270°	25.2	1.34	4.9	28.1	9.4
315°	26.6	1.42	3.3	28.8	11.7

table 29.2. Some values for the flights of fig. 29.3.

The wind speed at ground level $Z=0$ is 2 m/s, at $Z=10$ m: 3 m/s and at the launching point: 2.18 m/s. Values for the "modified initial conditions", V'_0 , U'_0 , Ψ'_0 for these flights are listed in table 29.2 together with values for the maximum elevation Z_2 and the distance of the landing point D. In all 8 cases the average wind speed of about 2.5 m/s encountered by the boomerang spoils the return performance. Two main effects are visible: 1^o the boomerang is transported downwind, and 2^o the maximum height Z_2 increases with increasing values of $\cos(\beta-\varphi)$, varying between 6.8 m for the throw downwind to 12.6 m for the throw upwind.

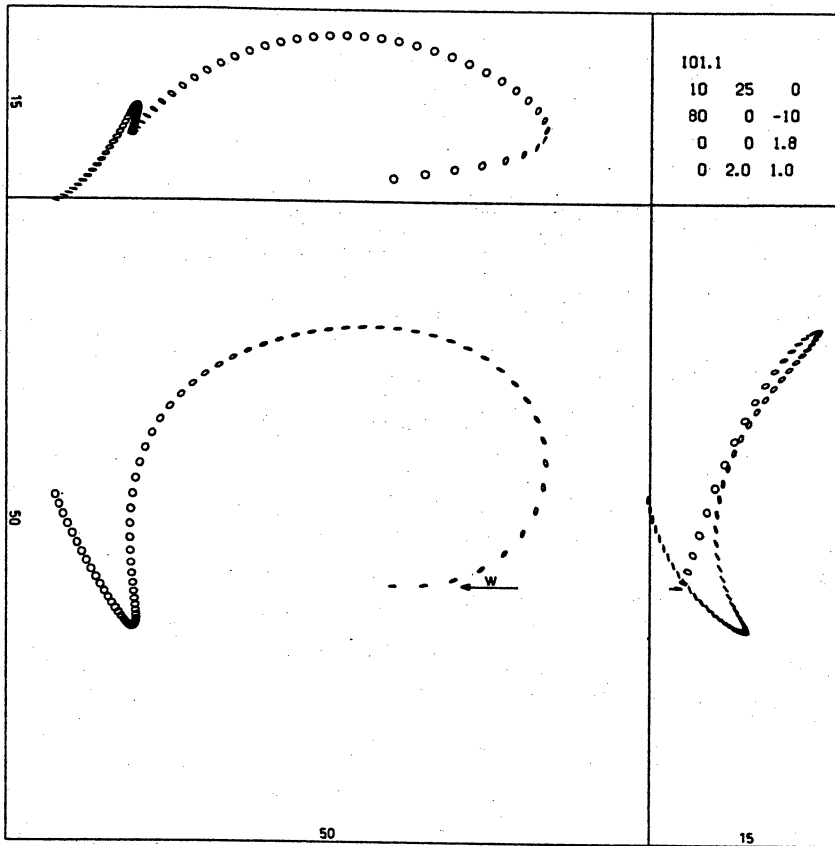


fig. 29.3a

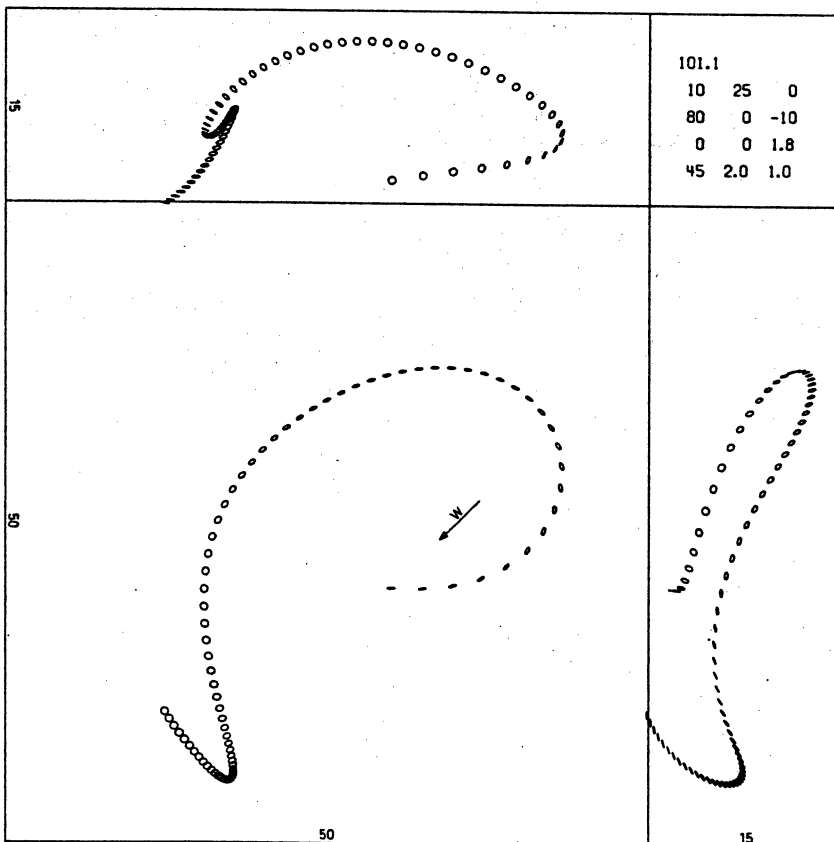


fig. 29.3b